# A Unified Capacity Analysis for Wireless Systems With Joint Multiuser Scheduling and Antenna Diversity in Nakagami Fading Channels

Chiung-Jang Chen and Li-Chun Wang, Member, IEEE

Abstract-In this paper, we present a cross-layer analytical framework to jointly investigate antenna diversity and multiuser scheduling under the generalized Nakagami fading channels. We derive a unified capacity formula for the multiuser scheduling system with different multiple-input multiple-output antenna schemes, including: 1) selective transmission/selective combining (ST/SC); 2) maximum ratio transmission/maximum ratio combining (MRT/MRC); 3) ST/MRC; and 4) space-time block codes (STBC). Our analytical results lead to the following four observations regarding the interplay of multiuser scheduling and antenna diversity. First, the higher the Nakagami fading parameter, the lower the multiuser diversity gain for all the considered antenna schemes. Second, from the standpoint of multiuser scheduling, the multiple antennas with the ST/SC method can be viewed as virtual users to amplify multiuser diversity order. Third, the boosted array gain of the MRT/MRC scheme can compensate the detrimental impact of the reduced amount of fading gain on multiuser scheduling, thereby resulting in greater capacity than the ST/SC method. Last, employing the STBC scheme together with multiuser diversity may cause capacity loss due to the reduced amount of fading gain, but without the supplement of array gain.

*Index Terms*—Diversity methods, fading channels, multipleinput multiple-output (MIMO) systems, scheduling.

## I. INTRODUCTION

**D** IVERSITY techniques are known as the effective means to cope with fading in wireless channels. The fundamental philosophy behind diversity techniques is to produce independent replicas of the desired signal over fading channels so that the receiver can utilize the multiple faded copies to restore the original signal with higher reliability. For a long time, the concept of diversity has been substantiated in various forms, such as spatial diversity, temporal diversity and frequency diversity [1].

In point-to-point communication links, multiple antenna systems have been widely used to achieve spatial (antenna) diversity. On the one hand, when multiple antennas are employed at the receiver, selective combining (SC), or maximum ratio

C.-J. Chen is with Chunghwa Telecom Laboratories, Taoyuan City 330, Taiwan, R.O.C. (e-mail: cjch@cht.com.tw).

L.-C. Wang is with the Department of Communication Engineering, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C. (e-mail: lichun@cc.nctu.edu.tw).

Digital Object Identifier 10.1109/TCOMM.2005.863778

combining (MRC) schemes are usually used to provide antenna diversity gain. On the other hand, with multiple antennas employed at the transmitter, techniques like selective transmission (ST) and maximum ratio transmission (MRT) can be also used to yield antenna diversity gain. Furthermore, joint transmit and receive diversity schemes were also proposed to improve link quality over the multiple-input multiple-output (MIMO) fading channel, e.g., ST/MRC [2] and MRT/MRC [3], [4]. More recently, the space–time block code (STBC) methods were introduced to deliver antenna diversity gain without requiring the prior channel knowledge at the transmitter [5], [6].

In point-to-multipoint communication systems, another type of diversity, called the multiuser diversity, can be exploited to improve spectral efficiency [7], [8]. This kind of diversity can be explained as an analogy of the water-filling principle across multiple users: higher system spectral efficiency can be attained by pouring more resources to the user with better channel quality. A proper scheduling algorithm is the key to extract the multiuser diversity inherent in the multiuser system [9], [10], [13]. Some current industrial standards, such as the IS-856 [11] and the 3GPP R5 [12], have adopted scheduling techniques to enhance spectral efficiency for delay-insensitive data services.

Generally speaking, scheduling is a media-access control (MAC) layer technique to deliver multiuser diversity gain by taking advantage of independent channel variations among user population. By contrast, antenna diversity is a physical layer approach to offer reliable transmissions with the major goal of mitigating channel fading. In a multiuser system where a channel-aware scheduling algorithm arranges transmissions based on the link quality of multiple users, various antenna diversity schemes may provide different effective link statistics and ultimately may lead to distinct capacity results with the effect of scheduling. Thus, the cross-layer interaction between antenna diversity and multiuser scheduling has attracted a lot of attention from the research community.

The literature survey with the joint effect of multiuser scheduling and antenna diversity is discussed as follows. For the multiple-input single-output (MISO) fading channel, the authors in [13] proposed an opportunistic beamforming scheme to approach the capacity supported by the coherent MRT method by exploiting multiuser diversity. In [14], a simulation study was conducted to compare the network throughput achieved by the ST and the STBC methods in the presence of scheduling. For the single-input multiple-output (SIMO) fading channel, a semianalytical result in terms of spectral efficiency with L-order receive MRC and K-order multiuser diversity was derived in

Paper approved by Y. Fang, the Editor for Wireless Networks of the IEEE Communications Society. Manuscript received April 2, 2004; revised June 6, 2005. This work was supported by the National Science Council and the Program of Promoting Excellence of University of Ministry of Education, Taiwan, R.O.C., under Contract EX91-E-FA06-4-4, Contract NSC92-2213-E-009-097. This paper was presented in part at the IEEE International Conference on Communications, Paris, France, June 2004.

[15]. Moreover, the authors in [16] evaluated the impact of multiuser scheduling on the STBC technique in terms of the cumulative distribution function (CDF) of the effective receive signal-to-noise ratio (SNR). Unlike [13]–[16] where specific antenna diversity methods are considered, [17] and [18] adopted an information theoretic approach to investigate the interplay of antenna diversity and multiuser diversity. We note that most of the previous works either assume a Rayleigh fading channel [14]–[18] or consider only limited antenna diversity schemes [13]–[16].

In this paper, we present a unified analytical framework to investigate the capacity of wireless systems with joint antenna diversity and multiuser scheduling in the generalized Nakagami fading channel. We first derive a closed-form expression for the system capacity integrating the performance improvements contributed from three different domains: fading channel characteristics, user population and multiple antenna systems. Specifically, under the Nakagami fading channel, we derive the capacity of a multiuser scheduling system with the following four MIMO antenna schemes in the downlink scenario: 1) selective transmission/selective combining (ST/SC), standing for the ST and SC schemes being used at the transmitter and the receiver, respectively; 2) maximum ratio transmission/maximum ratio combining (MRT/MRC); 3) selective transmission/maximum ratio combining (ST/MRC); and 4) space-time block codes (STBC). We further express the unified capacity formula in terms of three key parameters: array gain; amount of fading gain; and selection order to provide insights into the interaction of antenna diversity and multiuser scheduling. The values of these parameters corresponding to all the studied antenna schemes are also tabulated. Using the table together with an asymptotic upper bound of system capacity, we offer a simple and unified view on the tradeoff between complexity and performance, which can be useful in implementing practical antenna schemes on top of the multiuser scheduling system.

The rest of this paper is organized as follows. In Section II, we describe the channel model and introduce two lemmas for later use. In Section III, we derive a closed-form formula for the capacity of the single-antenna system with multiuser scheduling. In Section IV, we extend the analysis by incorporating the MIMO antenna system. In Section V, we suggest a set of parameter transformation to elaborate the joint effect of antenna diversity and multiuser scheduling on system capacity. Section VI presents numerical results. In Section VII, we provide our concluding remarks.

#### II. CHANNEL MODEL

Fig. 1 shows a wireless system with a base station serving K downlink users. To begin with, we first assume that the base station and each user have only one single antenna. Let  $h_k$  be the channel gain between the base station and user k, and let  $\sigma_n^2$  be the thermal noise power. We assume that the base station transmits at a fixed power level  $P_t$  at all times without power control. Thus,  $\gamma_k = P_t |h_k|^2 / \sigma_n^2$  denotes the received instantaneous SNR of user k. We assume that each link is subject to



Fig. 1. Multiuser scheduling system with  $N_t$  antennas at the transmitter and  $N_r$  antennas at each receiver.

independent Nakagami fading with a common Nakagami fading parameter m.<sup>1</sup> Then, the probability density function (PDF) of the received SNR for user k is [1]

$$p_{\gamma_k}(\gamma) = \left(\frac{m}{\Omega_k}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-\frac{m\gamma}{\Omega_k}\right), \quad \gamma > 0 \quad (1)$$

where  $\Omega_k$  is the average received SNR, and  $\Gamma(\cdot)$  is the Gamma function defined by

$$\Gamma(m) = \int_0^\infty t^{m-1} e^{-t} dt.$$
 (2)

When m = 1, the Nakagami fading channel is identical to the Rayleigh fading channel. For m > 1, a line-of-sight or a specular component exists. As  $m \to \infty$ , the Nakagami channel approaches the AWGN channel.

To ease notation, we denote  $X \sim \mathcal{G}(\alpha, \beta)$  as a Gamma distributed random variable with parameters  $\alpha$  and  $\beta$ . Then, the PDF of X is given as [19]

$$p_X(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}, \quad x > 0.$$
(3)

Furthermore, the cumulative distribution function (CDF) of X can be expressed by

$$P_X(x) = \widetilde{\Gamma}(\alpha, \beta x) \tag{4}$$

where  $\widetilde{\Gamma}(\cdot, \cdot)$  is the incomplete Gamma function defined by

$$\widetilde{\Gamma}(a,x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt.$$
(5)

Based on the above notation, the distribution of  $\gamma_k$  in the Nakagami fading channel model can be represented by  $\gamma_k \sim \mathcal{G}(m, m/\Omega_k)$ .

Next, we introduce two lemmas regarding the properties of Gamma random variables, which will be used in Section IV.

Lemma 1: Let  $X_1, X_2, \ldots, X_K$  be independent Gamma random variables with parameters  $\alpha_k$  and  $\beta$ , respectively. Let Y be the random variable given by  $Y = X_1 + \cdots + X_K$ . Then we have

$$Y \sim \mathcal{G}\left(\sum_{k=1}^{K} \alpha_k, \beta\right).$$
(6)

*Proof:* See [19]. 
$$\Box$$

<sup>1</sup>While different users may experience different levels of fading, the system capacity obtained based on the assumption of a common Nakagami fading parameter can be approximately thought of as the result that averages out the cases of different  $m_k$  with a common representative value m.

1

*Lemma 2:* Let X be a Gamma random variable with parameters  $\alpha$  and  $\beta$ . Let Y be the random variable given by Y = cX, c > 0. Then we have

$$Y \sim \mathcal{G}\left(\alpha, \frac{\beta}{c}\right). \tag{7}$$

*Proof:* The proof is completed by using a simple variable transformation  $f_Y(y) = f_X(y/c)/c \sim \mathcal{G}(\alpha, \beta/c)$ .

## III. CAPACITY WITH MULTIUSER SCHEDULING

In this section, we derive the capacity expression for a multiuser scheduling system with the single-input single-output (SISO) antenna. We assume that each user can track its channel status via, say, a downlink common pilot and correctly feed back to the base station without delay. In turn, similar to the scheduling mechanism in the modern cellular standard IS-856 [11], the base station determines to service a target user in every time slot with its full transmit power  $P_t$ . Here, we assume that the channel variation remains constant over one time slot, but independently varies in different time slots.

### A. Capacity Analysis

The scheduling policy considered in this paper is to select a target user  $k^*$  according to the following rule:<sup>2</sup>

$$k^* = \arg\max_k \frac{\gamma_k}{\tilde{\gamma_k}} \tag{8}$$

where  $\tilde{\gamma}_k$  is the average SNR of user k measured over a past window of length  $t_c$ . Note that  $\tilde{\gamma}_k$  is introduced in the denominator to maintain the long-term fairness. Under the assumption that all users have infinite data to send and  $t_c \to \infty$ ,  $\tilde{\gamma}_k$ is equivalent to  $\Omega_k$ , approximately [20]. To focus on the impact of small-scale channel fading on multiuser diversity gain, we let  $\Omega_k = \Omega$  for all k in the remainder of the paper.<sup>3</sup> Then, the scheduling rule in (8) is reduced to

$$k^* = \arg\max_k \gamma_k. \tag{9}$$

Let  $f_{\gamma_{\max}}(\alpha, \beta, K)$  be the PDF of  $\max(\gamma_1, \ldots, \gamma_K)$ , where  $\{\gamma_k\}_{k=1}^K$  denotes a set of K independent and identically distributed (i.i.d.) Gamma variates with parameters  $\alpha$  and  $\beta$ . From [27], we have

$$f_{\gamma_{\max}}(\alpha,\beta,K) = Kp_{\gamma}(\gamma) \left[P_{\gamma}(\gamma)\right]^{K-1}$$
(10)

where  $p_{\gamma}(\gamma)$  and  $P_{\gamma}(\gamma)$  are defined in (3) and (4). Let  $f_{\text{link}}^{k^*}(\gamma)$  be the conditional PDF of the received SNR, given that the connection between the base station and user  $k^*$  is established. Clearly, according to (9), we have  $f_{\text{link}}^{k^*}(\gamma) = f_{\gamma_{\text{max}}}(\alpha, \beta, K)$  for the SISO case. Once the target user is determined, adaptive modulation is applied to transmit as many information bits as

<sup>2</sup>The rule presented in this paper is slightly different from the proportional fair scheduling introduced in [9] and [13]. If the data rate supported by the channel is proportional to SNR, both criteria are the same.

possible. From [21], the link capacity between the base station and the target user  $k^*$  then can be written as

$$C_{\text{link}}^{k^*} = \int_0^\infty \log_2(1+\gamma) f_{\text{link}}^{k^*}(\gamma) d\gamma.$$
(11)

Note that the capacity expression in (11) is normalized to the bandwidth, thereby having the unit of bits/seconds/Hertz (b/s/Hz).

With link capacity, now we proceed to derive the system capacity. We define the system capacity as the sum of the link capacity delivered to each user on average. Let  $p_{k^*}$  be the average probability of user  $k^*$  receiving services from the base station. Then the system capacity is expressed by

$$C_{\rm sys} = \sum_{k^*=1}^{K} C_{\rm link}^{k^*} p_{k^*}.$$
 (12)

Note that the scheduling policy in (9) is fair, in the sense that each user can have equal probability (or time) to receive services from the base station in the long run. Thus, we have  $p_{k^*} = 1/K$ . Furthermore, since the channel variations among multiple users are assumed to be mutually independent, the system capacity in (12) can be written as

$$C_{\rm sys} = \frac{1}{K} \sum_{k^*=1}^{K} C_{\rm link}^{k^*} = C_{\rm link}^{k^*}.$$
 (13)

Equation (13) implies that system capacity is equal to the conditional link capacity when a specific user is chosen from K users. In other words, the capacity gain is achieved by providing the system with multiuser diversity, i.e., more selections of independent channel variations experienced by multiple users. Finally, substituting (10) and (11) in (13) yields

$$C_{\rm sys} = \frac{K\beta^{\alpha}\log_2(e)}{\Gamma(\alpha)} \times \int_0^\infty \ln(1+\gamma) \left[\widetilde{\Gamma}(\alpha,\beta\gamma)\right]^{K-1} \cdot \gamma^{\alpha-1} e^{-\beta\gamma} d\gamma.$$
(14)

When the parameter  $\alpha$  is an integer value, (14) can be evaluated in terms of finite series expansions by (15) at the bottom of the next page (see Appendix A), where  $a_i^k$  for  $0 \le i \le k(\alpha - 1)$ can be recursively calculated by

$$a_{0}^{k} = 1, \quad a_{1}^{k} = k,$$

$$a_{i}^{k} = \frac{1}{i} \sum_{n=1}^{\min(i, \alpha - 1)} \frac{n(k+1) - i}{n!} a_{i-n}^{k}, \text{ for } 2 \le i < k(\alpha - 1)$$

$$a_{i}^{k} = \frac{1}{[(\alpha - 1)!]^{k}}, \quad \text{for } i = k(\alpha - 1)$$
(16)

and  $E_1(\cdot)$  is the exponential integral function of the first kind defined as [22]

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt.$$
 (17)

As a result, for integer values of Nakagami fading parameter m, we have

$$C_{\rm sys} = C_1\left(m, \frac{m}{\Omega}, K\right). \tag{18}$$

<sup>&</sup>lt;sup>3</sup>The analysis for the general  $\Omega_k$  is similar to the  $\Omega_k = \Omega$  case, once one uses the proportional fair principle [13] or normalization technique [34] for the scheduling algorithm to compensate different  $\Omega_k$  due to users' near-far locations.

Some special cases of (18) can be further derived as follows.

• For the Rayleigh fading case (m = 1) with K-fold multiuser diversity, (18) is simplified to

$$C_{\rm sys} = K \log_2(e) \\ \times \sum_{k=0}^{K-1} (-1)^k \binom{K-1}{k} \frac{e^{(k+1)/\Omega}}{k+1} \cdot E_1\left(\frac{k+1}{\Omega}\right).$$
(19)

It is noted that (19) has an equivalent mathematical expression with [26, eq. 44]. This implies that the multiuser diversity in the multiuser SISO system is essentially analogous to the selection diversity in the single-user SIMO system.

- For the single-user case K = 1, multiuser diversity gain vanishes and (18) reduces to (20), shown at the bottom of the page, which is identical to the link capacity of the Nakagami fading channel with optimal rate control [23], [24].
- For m = 1 and K = 1, (18) is simply reduced to the link capacity of the Rayleigh fading channel as shown in [25], [26]

$$C_{\rm sys} = \log_2(e)e^{1/\Omega}E_1\left(\frac{1}{\Omega}\right).$$
 (21)

When m is not restricted to an integer, (14) can be also efficiently computed with the help of the orthogonal Laguerre polynomial as follows:

$$C_{\text{sys}} \simeq \frac{K \log_2(e)}{\Gamma(\alpha)} \sum_{i=1}^{N_L} w_i \ln\left(1 + \frac{z_i}{\beta}\right) \left[\widetilde{\Gamma}(\alpha, z_i)\right]^{K-1} z_i^{\alpha-1} \\ \triangleq C_2(\alpha, \beta, K)$$
(22)

where  $w_i$  and  $z_i$  are the weights and zeros of the Laguerre polynomial [22], and  $N_L$  is the order of polynomial chosen to make the approximation error negligibly small. Consequently, for the general Nakagami-m fading environments, the capacity of the multiuser scheduling system with K-fold multiuser diversity can be expressed by

$$C_{\rm sys} = C_2\left(m, \frac{m}{\Omega}, K\right). \tag{23}$$

With (18) or (23), we have established a closed-form capacity expression with the contribution of K-fold multiuser diversity



Fig. 2. Impact of Nakagami-m channel fading on the capacity of the multiuser scheduling system.

and the impact of Nakagami-m channel fading. Next, we give a numerical example using (18).

#### B. A Numerical Example

Fig. 2 shows the impact of Nakagami fading on the capacity of the multiuser scheduling system. The mean SNR is set to one, i.e.,  $\Omega = 0$  dB, in this example. The information capacity of the additive white Gaussian noise (AWGN) channel with the same mean SNR is also plotted for comparison. From this figure, one can see that the capacity for K = 1 is always lower than that of the AWGN channel. Moreover, a more scattering environment with a smaller value of m yields lower capacity when K = 1. However, the story becomes totally different when more than one user resides in the system. One can find that in the presence of multiuser diversity ( $K \ge 2$ ), the capacity of the multiuser scheduling system is the highest for the Rayleigh fading channel (m = 1), and decreases as m increases. When  $m \to \infty$ , the

$$C_{\text{sys}} = \frac{K \log_2(e)}{(\alpha - 1)!} \sum_{k=0}^{K-1} (-1)^k \binom{K-1}{k} e^{(k+1)\beta} \sum_{i=0}^{k(\alpha - 1)} a_i^k \beta^{\alpha + i} \cdot (\alpha + i - 1)! \\ \times \sum_{j=1}^{\alpha + i} \frac{(-1)^{\alpha + i - j}}{(\alpha + i - j)!} \left[ \frac{1}{(k+1)\beta} \right]^j \left\{ E_1 \left( (k+1)\beta \right) - e^{-(k+1)\beta} \sum_{l=0}^{\alpha + i - j - 1} (-1)^l l! \left[ \frac{1}{(k+1)\beta} \right]^{l+1} \right\} \\ \triangleq C_1(\alpha, \beta, K)$$
(15)

$$C_{\rm sys} = \log_2(e) \sum_{j=1}^m \frac{(-1)^{m-j}}{(m-j)!} \left(\frac{m}{\Omega}\right)^{m-j} \left[ e^{m/\Omega} E_1\left(\frac{m}{\Omega}\right) - \sum_{l=0}^{m-j-1} (-1)^l l! \left(\frac{\Omega}{m}\right)^{l+1} \right]$$
(20)

channel becomes the AWGN channel and no multiuser diversity gain can be exploited any more. We note that Fig. 2 focuses on the effect of small-scale channel fading on the capacity of the multiuser scheduling system assuming equal mean SNR for all users. Whether a more specular environment with a larger value of m will simultaneously incur higher mean path gain or higher mean SNR requires the knowledge of the correlation between mean path gain and m. It is beyond the scope of this paper to study the composite effect of channel fading and propagation loss on the resulting capacity of the multiuser scheduling system.

As shown in Fig. 2, a more scattering fading environment is beneficial for the considered multiuser scheduling system, since the larger channel variations enable the scheduler to arrange transmissions at higher peaks of channel fading. On the other hand, considering the possibly contradictory goal of stabilizing the fading link for most antenna diversity techniques, one should be careful in employing the antenna diversity schemes on top of the multiuser scheduling system. In the next section, we will examine the capacity achieved in the multiuser scheduling system with some practical MIMO antenna diversity schemes.

# IV. CAPACITY WITH JOINT MULTIUSER SCHEDULING AND ANTENNA DIVERSITY

In this section, we extend our analysis to incorporate some practical MIMO antenna diversity schemes with multiuser scheduling. To this end,  $N_t$  transmit antennas and  $N_r$  receive antennas are employed at the base station and each user, respectively. We assume that the links between each pair of transmit and receive antennas are subject to i.i.d. Nakagami fading. Then the channel between the base station and user k can be modeled by an  $N_r \times N_t$  matrix  $\mathbf{H}_k = [h_{ij}^{(k)}]$ , where each  $P_t |h_{ij}^{(k)}|^2 / \sigma_n^2$  is Gamma distributed as defined in (1). In the following, we consider four antenna diversity schemes, including: 1) ST/SC; 2) MRT/MRC; 3) ST/MRC; and 4) STBCs. All four considered antenna schemes are capable of delivering full antenna diversity order over the MIMO channels [1]–[3], [5]. For a fair comparison, the total transmit power across all  $N_t$  antennas is constrained to the same level for all the antenna schemes.

#### A. ST/SC Scheme

Assume that orthogonal pilot signals transmit from  $N_t$  spatially separated antennas at the base station. By monitoring the pilot signals, each user can distinguish the link with the strongest SNR from  $N_t N_r$  possible transmit and receive antenna pairs at any time slot. Collecting feedback from all K users, the base station can determine the target user and the associated transmit antenna radiating the best link quality [1], [14]. Under this policy, the scheduling rule can be mathematically expressed as

$$k^* = \arg\max_k \arg\max_{(i,j)} |h_{ij}^{(k)}|^2.$$
(24)

Thus, we have  $f_{\text{link}}^{k^*}(\gamma) = f_{\gamma_{\text{max}}}(\alpha, \beta, KN_tN_r)$  for the ST/SC scheme, where  $f_{\gamma_{\text{max}}}(\cdot, \cdot, \cdot)$  is defined in (10). Comparing the conditional PDF  $f_{\text{link}}^{k^*}(\gamma)$  associated with the ST/SC scheme and that of the SISO case, we obtain the system capacity with

joint multiuser scheduling and the ST/SC antenna scheme in Nakagami fading channels, as follows:

$$C_{\text{st-sc}} = C_2\left(m, \frac{m}{\Omega}, KN_t N_r\right).$$
(25)

Expression (25) can be interpreted from two folds. On the one hand, from a multiuser system point of view, the base station can see total  $KN_r$  antennas at the receiving end, rather than only  $N_r$ . The additional  $K(N_r - 1)$  virtual antennas from other users can be exploited to improve system capacity. On the other hand, the multiple antennas in the ST/SC scheme can be viewed as virtual users to increase the multiuser diversity order for the considered scheduling algorithm. Thus, we can take a broader view to define the selection order S as the size of a set with S i.i.d. Gamma random variables, which are provided by the multiple users and/or multiple antennas in the multiuser MIMO system. Consequently, the capacity improvement achieved by the ST/SC MIMO scheme with multiuser scheduling can be explained by the fact that the selection order is indeed increased from S = K to  $S = KN_tN_r$ .

#### B. MRT/MRC Scheme

In [3], the authors proposed an MRT/MRC scheme to deliver full antenna diversity over the MIMO channel, which was later analyzed in [4] under the Rayleigh fading channel in more detail. Given a known channel matrix, this method can maximize the received SNR by applying the specific beamforming weight  $\mathbf{w}_t$  at the transmitter and combining weight  $\mathbf{w}_r$  at the receiver. It was shown that by setting  $\mathbf{w}_t$  and  $\mathbf{w}_r$  to be the principal right and left singular vectors of the channel matrix, respectively, the optimal received SNR can be attained with the effective output SNR  $\gamma_k = P_t \lambda_{\max}(\mathbf{H}_k^{\dagger} \mathbf{H}_k) / \sigma_n^2$ , where  $\mathbf{H}_k^{\dagger}$  is the transpose conjugate of  $\mathbf{H}_k$ , and  $\lambda_{\max}(\mathbf{H}_k^{\dagger} \mathbf{H}_k)$  is the maximum eigenvalue of  $\mathbf{H}_k^{\dagger} \mathbf{H}_k$ . Based on the effective output SNR  $\gamma_k$ , the decision rule for the scheduler is

$$k^* = \arg\max_k \, \lambda_{\max}(\mathbf{H}_k^{\dagger} \mathbf{H}_k). \tag{26}$$

Note that the effective output SNR  $\gamma_k$  is a random variable depending on different realizations of the random channel matrix. Recently, the exact distribution of  $\lambda_{\max}(\mathbf{H}_k^{\dagger}\mathbf{H}_k)$  for Rayleigh channels is given by [4]

$$f_{\lambda_{\max}}(\lambda) = \sum_{i=1}^{N_r} \sum_{j=N_t-N_r}^{(N_t+N_r)i-2i^2} \frac{d_{ij}i^{j+1}\lambda^j e^{-i\lambda}}{j!}$$
(27)

where  $d_{ij}$  is the associated coefficient determined by different combinations of  $N_t$  and  $N_r$ . As one can see from (27), it is not easy to derive the conditional PDF  $f_{\text{link}}^{k^*}(\gamma)$  directly based on (27). However, for generalized Nakagami fading channels, we can use the following inequality [28]:

$$\frac{\|\mathbf{H}_k\|_F^2}{N} \le \lambda_{\max}(\mathbf{H}_k^{\dagger}\mathbf{H}_k) \le \|\mathbf{H}_k\|_F^2$$
(28)

to obtain the lower and the upper bound of system capacity. Note that in (28),  $N = \min(N_t, N_r)$  denotes the minimum number of  $N_t$  and  $N_r$ , and  $||\mathbf{H}_k||_F$  is the Frobenius matrix norm

MIMO antennaschemes	System capacity	Array $gain(a)$	AF gain $(f)$	Selection order $(S)$
$C_{ m siso},(1,1)$	$C_2\left(m, \frac{m}{\Omega}, K\right)$	1	1	K
$C_{\rm sc}, (1, N_r)$	$C_2\left(m, \frac{m}{\Omega}, KN_r\right)$	1	1	$KN_r$
$C_{\mathrm{mrc}}, (1, N_r)$	$C_2\left(mN_r, \frac{m}{\Omega}, K\right)$	$N_r$	$1/N_r$	K
$C_{\mathrm{st}},(N_t,1)$	$C_2\left(m, \frac{m}{\Omega}, KN_t\right)$	1	1	$KN_t$
$C_{\mathrm{mrt}},(N_t,1)$	$C_2\left(mN_t, \frac{m}{\Omega}, K\right)$	$N_t$	$1/N_t$	K
$C_{\text{st-sc}}, (N_t, N_r)$	$C_2\left(m,\frac{m}{\Omega},KN_tN_r\right)$	1	1	$KN_tN_r$
$C_{\text{st-mrc}}, (N_t, N_r)$	$C_2\left(mN_r, \frac{m}{\Omega}, KN_t\right)$	$N_r$	$1/N_r$	$KN_t$
$C_{\mathrm{mrt-mrc}}^{\mathrm{u}}, (N_t, N_r)$	$C_2\left(mN_tN_r, \frac{m}{\Omega}, K\right)$	$N_t N_r$	$1/N_t N_r$	K
$C^{\mathbf{l}}_{\mathbf{mrt-mrc}}, (N_t, N_r)$	$C_2\left(mN_tN_r, \frac{mN}{\Omega}, K\right)$	$N_t N_r / N$	$1/N_tN_r$	K
$C_{\mathrm{stbc}}, (N_t, N_r)$	$C_2\left(mN_tN_r, \frac{mN_t}{\Omega}, K\right)$	$N_r$	$1/N_t N_r$	K

 TABLE I
 I

 CAPACITY OF THE MULTIUSER SCHEDULING SYSTEM WITH DIFFERENT ANTENNA DIVERSITY SCHEMES

with  $||\mathbf{H}_k||_F^2 = \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |h_{ij}^{(k)}|^2$ . Thus, applying *Lemmas 1* and 2 to (28), one can use  $f_{\text{link}}^{k^*}(\gamma) = f_{\gamma_{\max}}(\alpha N_t N_r, \beta N, K)$  and  $f_{\text{link}}^{k^*}(\gamma) = f_{\gamma_{\max}}(\alpha N_t N_r, \beta, K)$  to further derive the lower and upper bounds for the system capacity with the MRT/MRC scheme, respectively. By comparing these conditional PDFs with that of the SISO case, we reach

$$C_2\left(mN_tN_r, \frac{mN}{\Omega}, K\right) \leq C_{\text{mrt-mrc}} \leq C_2\left(mN_tN_r, \frac{m}{\Omega}, K\right)$$
(29)

where  $C_{mrt-mrc}$  is the system capacity achieved by the MRT/MRC scheme with multiuser scheduling.

In the cases of SIMO ( $N_t = 1$ ) and MISO ( $N_r = 1$ ), the channel matrix is reduced to a rank-one vector. Under such conditions, the achieved system capacity in (29) is equal to the upper-bound expression, i.e.,  $C_{\rm mrc} = C_2(mN_r, m/\Omega, K)$ for the receive MRC, and  $C_{\rm mrt} = C_2(mN_t, m/\Omega, K)$  for the transmit MRT, respectively.

### C. ST/MRC Scheme

Here, we study a hybrid scheme, which implements the ST at the transmitter and the MRC at the receiver over the MIMO channel [2]. With the MRC method used at the receiver, the effective SNR at the *k*th user's combiner output, with respect to the *j*th transmit antenna, can be written as  $\gamma_k = P_t \sum_{i=1}^{N_r} |h_{ij}^{(k)}|^2 / \sigma_n^2$  [1]. Similarly, the base station gathers the effective SNR of all users and selects the target user according to the following criterion:

$$k^* = \arg\max_k \arg\max_j \sum_{i=1}^{N_r} |h_{ij}^{(k)}|^2.$$
(30)

Applying Lemma 1 to (30), we have  $f_{\text{link}}^{k^*}(\gamma) = f_{\gamma_{\text{max}}}(\alpha N_r, \beta, KN_t)$  for the ST/MRC scheme. Accordingly, the capacity of the multiuser scheduling system with the ST/MRC scheme is described as follows:

$$C_{\text{st-mrc}} = C_2\left(mN_r, \frac{m}{\Omega}, KN_t\right).$$
(31)

# D. STBC Scheme

STBCs pertain to another category of antenna schemes to provide antenna diversity gain without requiring prior channel knowledge at the transmitter. In this paper, we focus on the STBC with orthogonal structures, which was introduced in [5] and developed more generally in [6]. Through coding over time as well as transmit antennas, the orthogonal STBC technique can also deliver full antenna diversity order by using simple linear processing at the receiver. From [16] and [29], the effective SNR of user k at the output of the STBC decoder is given by  $\gamma_k = P_t/N_t \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |h_{ij}^{(k)}|^2 / \sigma_n^2$ . Thus, the corresponding selection rule for the base station scheduler is

$$k^* = \arg\max_k \frac{1}{N_t} \sum_{i=1}^{N_t} \sum_{j=1}^{N_t} |h_{ij}^{(k)}|^2.$$
(32)

Again, applying *Lemmas 1* and 2 to (32), we have  $f_{\text{link}}^{k^*} = f_{\gamma_{\text{max}}}(\alpha N_t N_r, \beta N_t, K)$  for the STBC method. Consequently, the capacity of the multiuser scheduling system with the orthogonal STBC can be expressed as

$$C_{\rm stbc} = C_2 \left( m N_t N_r, \frac{m N_t}{\Omega}, K \right).$$
(33)

Notice that the system capacity achieved with joint multiuser scheduling and the orthogonal STBC will, in general, be lower than (33). The reason follows from the fact that only the STBCs with full code rate can support the capacity promised by (33), while the full code rate STBCs are available for a limited number of transmit antennas and signal constellations [6]. Nevertheless, we still can use (33) to evaluate the impact of applying the STBC scheme on top of the multiuser scheduling system, since it represents an optimistic performance upper bound.

# E. Discussions

Table I summarizes the system capacity achieved with all the aforementioned antenna diversity schemes in the considered multiuser scheduling system. Comparing  $C_{\rm st}$  with  $C_{\rm sc}$  and  $C_{\rm mrt}$  with  $C_{\rm mrc}$  in Table I, one can find that the transmit and receive diversity methods can achieve identical system capacity,

provided that  $N_t$  in the MISO scheme is the same as  $N_r$  in the SIMO scheme. However, the practical considerations for implementing these antenna schemes in the multiuser scheduling system are quite different. First of all, the transmit methods have the potential advantage of relieving computation burdens for user terminals. Moreover, the cost (benefit) of adding one more antenna at the base station for the transmit methods can be amortized (shared) by multiple users. Next, the transmit methods generally require additional pilot signals, since the receiver would rely on them to estimate the SNR from each corresponding transmit antenna. Finally, the required amount of signaling in the feedback channel is usually larger for the transmit methods than the receive methods. For example, as compared with the receive SC method, using the transmit ST scheme requires the terminal to send back not only the SNR of the strongest channel, but also the signaling to indicate the best serving antenna. The transmit MRT scheme needs even a larger amount of overheads in the feedback channel, because the channel magnitude and phase for all transmit antennas are mandatory for the base station to perform the MRT scheme correctly.

# V. CAPACITY REVISITED: CHANGE OF COORDINATE PARAMETERS

So far, we have developed a unified capacity formula applicable to the wireless system with joint antenna diversity and multiuser scheduling. However, the closed-form expression (15) for integer  $\alpha$  and the approximate expression (22) for general  $\alpha$ may be too involved to clearly explain the interplay of multiuser scheduling and different antenna diversity schemes. For this reason, we further suggest a set of parameter transformations.

We first define the array gain a as

$$a \triangleq \frac{\mathrm{E}[\gamma_k]}{\Omega} = \frac{\frac{\alpha}{\beta}}{\Omega} \tag{34}$$

to account for the average increased level of received SNR, mainly due to coherent combining relative to the SISO case [30]. Note that in (34),  $\gamma_k \sim \mathcal{G}(\alpha, \beta)$  is the effective receive SNR at the combiner output for user k. Next, recall that the amount of fading (AF) associated with the PDF of  $\gamma_k$  is defined as [31]

$$\operatorname{AF}[\gamma_k] = \frac{\operatorname{Var}[\gamma_k]}{\left(\operatorname{E}[\gamma_k]\right)^2}$$
(35)

where  $\operatorname{Var}[\gamma_k]$  denotes the variance of  $\gamma_k$ . Following the definition of (35), one can easily find that  $\operatorname{AF}[\gamma_k] = 1/\alpha$  for  $\gamma_k \sim \mathcal{G}(\alpha, \beta)$ . The AF parameter can be viewed as a measure of the severity of fading, and is independent of the mean power for the Nakagami fading. Also, it can be a measure of the randomness of a random variable, namely, the higher the AF, the larger spread the fading distribution [32]. From (35), we further define the second parameter f, called the *amount of fading gain*, as

$$f \triangleq \frac{\operatorname{AF}[\gamma_k]}{\frac{1}{m}} = \frac{m}{\alpha}$$
(36)

to capture the relative randomness of  $\gamma_k$  with respect to the SISO case.

In terms of array gain a, AF gain f, and the previously defined selection order S, the system capacity then can be described by these three parameters as

$$C_3(a, f, S) = C_2\left(\frac{m}{f}, \frac{m}{af\Omega}, S\right).$$
(37)

Table I lists the values of a, f, and S corresponding to all the considered antenna schemes. Next, we introduce *Lemma 3* to further derive an analytical capacity upper bound.

*Lemma 3:* Let  $X_1, \ldots, X_S$  be i.i.d. random variables with common mean  $\mu$  and variance  $\sigma^2$ . Let  $Y = \max\{X_1, \ldots, X_S\}$ . Then the mean of the random variable Y is upper bounded by

$$\mathbf{E}[Y] \le \mu + \frac{(S-1)\sigma}{\sqrt{2S-1}}.$$
(38)

Proof: See [27].

Applying *Lemma 3* and Jensen's inequality to (37), one can obtain

$$C_{3}(a, f, S) \leq \log_{2} \left( 1 + a\Omega \left[ 1 + \frac{(S-1)}{\sqrt{2S-1}} \sqrt{\frac{f}{m}} \right] \right)$$
$$\simeq \log_{2} \left( 1 + a\Omega \left[ 1 + \sqrt{\frac{Sf}{2m}} \right] \right), \text{ for large } S. (39)$$

Essentially, (39) provides a simple capacity upper bound for effectively assessing the performance improvements attributed to fading characteristics, multiuser scheduling, and different antenna diversity schemes in the multiuser MIMO system. Referring to (39) together with Table I, we can make the following conjectures.

- 1) The higher value of Nakagami fading parameter m brings about a detrimental effect on the system capacity jointly achieved by the scheduling algorithm (9) and all the considered antenna schemes.
- 2) The ST/SC scheme can further improve the capacity of the multiuser scheduling system by amplifying multiuser diversity order. If a system already has possessed large selection order provided by inherent user population, it is expected that the attainable capacity gain with the additional ST/SC scheme may be somewhat limited.
- 3) Although the MRT/MRC scheme reduces the amount of fading gain, the combined effect of the increased array gain and the reduced amount of fading gain can still yield greater system capacity, as compared with the ST/SC method.
- 4) Employing the STBC via  $N_t$  transmit antennas can damp the channel fluctuations as though the Nakagami fading parameter changes from m to  $mN_t$ , but without the supplement of array gain. As a result, the STBC method may cause a negative impact on the capacity of the multiuser system with multiuser diversity. This is consistent with the observations in [13] and [16].

These conjectures will be further validated in the next section through numerical evaluations.

### VI. NUMERICAL RESULTS

In this section, we give some numerical examples using the derived unified capacity formula. The capacity result and the associated parameters are summarized in Table I, where  $(N_t, N_r)$ 



Fig. 3. Comparison of system capacity jointly achieved by multiuser scheduling and different antenna schemes.



Fig. 4. Impact of Nakagami-*m* channel fading on the attainable system capacity with joint multiuser scheduling and antenna diversity schemes.

represents the MIMO system with  $N_t$  transmit antennas and  $N_r$  receive antennas, and the superscripts u and l stand for the upper bound and the lower bound, respectively. In Figs. 3 and 4, we let  $\Omega = 0$  dB for numerical evaluations.

Fig. 3 compares the system capacity jointly achieved by multiuser scheduling and different antenna diversity schemes. One can see that the system capacity increases as the number of users K increases for Fig. 3(a)–(d). However, increasing the number of transmit antennas for the STBC method could degrade the capacity in the presence of multiuser scheduling ( $K \ge 2$ ), as shown in Fig. 3(d). Comparing Fig. 3(a)–(c), it is observed that the MRT/MRC method provides the highest capacity. We note that  $N_t = N_r$  is assumed for the ST/MRC case in Fig. 3(c). For the ST/SC method, the marginal benefit of adding additional antennas is more suppressed, as compared with the MRT/MRC method, especially when K is large. When comparing the capacity performance of Fig. 3(a), (b), and (d) over the MISO channel, we may interpret that the MRT schemes achieves the



Fig. 5. Impact of the operating mean SNR on the relative capacity gain, where the relative capacity gain is defined as the ratio of the system capacity achieved by joint multiuser scheduling and MIMO antenna schemes to that of K = 1 case.

highest capacity at the cost of the highest amount of feedback overhead. On the contrary, in a system with restricted feedback capability, applying the STBC method along with the traditional round-robin scheduling appears to be a feasible solution. We note that the comparison of the capacity results for the four different antenna schemes in Fig. 3 can be explained by using (39) along with Table I in Section V.

Fig. 4 shows the impact of Nakagami-m channel fading on the system capacity with joint multiuser scheduling and antenna diversity schemes. In this example, we let K = 32. One can see that the higher m causes a negative impact on the capacity for all the studied antenna schemes. When m increases from one to eight, the capacity drops 18.4% for the (4,1) MRT scheme, 37.7% for the (4,1) ST scheme, and 26.4% for the (4,1) STBC scheme, respectively. This implies that the ST/SC scheme is more sensitive to the condition of channel fading than the other schemes.

Fig. 5 demonstrates the impact of the operating mean SNR condition on the relative capacity gain. The relative capacity gain in this figure is defined as the ratio of the system capacity achieved by joint multiuser scheduling and MIMO antenna schemes to that achieved by the single-user case with only the same MIMO schemes. We note that the relative capacity gain is a dimensionless ratio with respect to the capacity of K = 1. We assume K = 32 and m = 1 in this example. As one can see from Fig. 5, the considered scheduling technique tends to bring about more relative capacity gain for the users with smaller  $\Omega$ . This implies that the users at the cell boundary can benefit more from the scheduling technique than those close to the base station. This property is quite useful, because in a system without downlink power control and soft handoff (such as IS-856), it is desirable to compensate the cell-edge users with more relative capacity gain.

$$C_{\rm sys} = \frac{K \log_2(e)}{(\alpha - 1)!} \sum_{k=0}^{K-1} \sum_{i=0}^{k(\alpha - 1)} (-1)^k \binom{K-1}{k}^k a_i^k \beta^{\alpha + i} e^{(k+1)\beta} \cdot (\alpha + i - 1)! \sum_{j=1}^{\alpha + i} \left[\frac{1}{(k+1)\beta}\right]^j \widetilde{\gamma} \left(j - \alpha - i, (k+1)\beta\right). \tag{42}$$

#### VII. CONCLUSIONS

In this paper, we have presented an analytical framework to investigate the capacity of wireless systems with joint antenna diversity and multiuser scheduling. We derive a unified capacity formula connecting K-fold multiuser diversity and some MIMO antenna diversity schemes over Nakagami-mfading channels. We further suggest three parameters: array gain; amount of fading gain; and selection order to assess the effect of applying different antenna diversity schemes on top of the multiuser scheduling system in the Nakagami fading channel. We also briefly discuss the system requirements for implementing the different antenna schemes in the multiuser scheduling system. The capacity results and related parameters for all the considered antenna diversity schemes are tabulated so that the tradeoff between performance and complexity can be effectively compared in a unified view. The joint effect of multiuser scheduling and the MIMO spatial multiplexing technique will be for future investigations.

# APPENDIX DERIVATION OF (15)

For integer values of  $\alpha$ , the PDF of (10) can be written as [33]

$$f_{\gamma_{\max}}(\gamma) = \frac{K}{(\alpha - 1)!} \sum_{k=0}^{K-1} \sum_{i=0}^{k(\alpha - 1)} (-1)^k \binom{K-1}{k} a_i^k \beta^{\alpha + i} \cdot e^{-(k+1)\beta\gamma} \gamma^{\alpha + i-1}$$
(40)

where the coefficient  $a_i^k$  is defined in (16). Substituting (40) to (13) and using the integral identity [26]

$$\int_{0}^{\infty} \ln(1+t)e^{-\mu t}t^{n-1}dt = (n-1)!e^{\mu}\sum_{j=1}^{n}\frac{\widetilde{\gamma}(j-n,\mu)}{\mu^{j}},$$
  
for  $n = 1, 2, 3, \dots$  (41)

we get (42), shown at the top of the page. Note that in (41) and (42),  $\tilde{\gamma}(\cdot, \cdot)$  is another form of incomplete gamma function, defined by

$$\widetilde{\gamma}(a,x) = \int_x^\infty t^{a-1} e^{-t} dt = \Gamma(a) \left[ 1 - \widetilde{\Gamma}(a,x) \right].$$
(43)

Finally, applying the identity [22, eq. 6.5.19]

$$\widetilde{\gamma}(-n,x) = \frac{(-1)^n}{n!} \left[ E_1(x) - e^{-x} \sum_{j=0}^{n-1} \frac{(-1)^j j!}{x^{j+1}} \right],$$
  
$$n = 0, 1, 2, \dots \quad (44)$$

to (42), we obtain (15).

#### ACKNOWLEDGMENT

The authors are grateful to Dr. M. Fang as well as the anonymous reviewers, who provided useful comments and remarks which helped them improve the quality of the paper.

#### REFERENCES

- [1] G. J. Stüber, *Principles of Mobile Communication*, 2nd ed. Amsterdam, The Netherlands: Kluwer, 2001.
- [2] S. Thoen, L. V. D. Perre, B. Gyselinckx, and M. Engels, "Performance analysis of combined transmit-SC/receive-MRC," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 5–8, Jan. 2001.
- [3] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.
- [4] P. A. Dighe, R. K. Mallik, and S. S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," *IEEE Trans. Commun.*, vol. 51, no. 4, pp. 694–703, Apr. 2003.
- [5] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
- [6] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.
- [7] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," in *Proc. Int. Conf. Commun.*, Jun. 1995, pp. 331–335.
- [8] D. N. C. Tse and S. Hanly, "Multiaccess fading channels—Part I: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Inf. Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.
- [9] A. Jalali, R. Padovani, and R. Pankaj, "Data throughput of CDMA-HDR, a high efficiency high data rate personal communication wireless system," *Proc. IEEE Veh. Technol. Conf.*, pp. 1854–1854, May 2000.
- [10] X. Liu, E. K. P. Chong, and N. B. Shroff, "Opportunistic transmission scheduling with resource-sharing constraints in wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 10, pp. 2053–2064, Oct. 2001.
- [11] Cdma2000: High-Rate Packet Data Air Interface Specification, TIA/EIA IS-856, 2000.
- [12] UTRA High Speed Downlink Packet Access, 3GPP TR25.950, 2001.
- [13] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.
- [14] A. G. Kogiantis, N. Joshi, and O. Sunay, "On transmit diversity and scheduling in wireless packet data," in *Proc. IEEE Int. Conf. Commun.*, vol. 8, Jun. 2001, pp. 2433–2437.
- [15] W. Wang, T. Ottosson, M. Sternad, A. Ahlen, and A. Svensson, "Impact of multiuser diversity and channel variability on adaptive OFDM," in *Proc. Veh. Technol. Conf.*, Orlando, FL, Oct. 2003.
- [16] R. Gozali, R. M. Buehrer, and B. D. Woerner, "The impact of multiuser diversity on space-time block coding," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 213–215, May 2003.
- [17] V. K. N. Lau, Y. Liu, and T. A. Chen, "The role of transmit diversity on wireless communications—Reverse link analysis with partial feedback," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2082–2090, Dec. 2002.
- [18] S. Borst and P. Whiting, "The use of diversity antennas in high-speed wireless systems: Capacity gains, fairness issues, multi-user scheduling," *Bell Labs Tech. Memo.*, 2001.
- [19] P. G. Hoel, S. C. Port, and C. J. Stone, *Introduction to Probability Theory*. Boston, MA: Houghton Mifflin, 1971.
- [20] J. M. Holtzman, "Asympotic analysis of proportional fair algorithm," in *Proc. IEEE Symp. PIMRC*, Oct. 2001, pp. F33–F37.
- [21] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 11, pp. 1986–1992, Nov. 1997.
- [22] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th ed. New York: Dover, 1970.
- [23] M.-S. Alouini and A. J. Goldsmith, "Capacity of Nakagami multipath fading channels," in *Proc. IEEE Veh. Technol. Conf.*, Phoenix, AZ, May 1997, pp. 358–362.
- [24] C. G. Günther, "Comment on estimate of channel capacity in Rayleigh fading environment," *IEEE Trans. Veh. Technol.*, vol. 45, no. 3, pp. 401–403, May 1996.
- [25] W. C. Y. Lee, "Estimate of channel capacity in Rayleigh fading environment," *IEEE Trans. Veh. Technol.*, vol. 39, no. 4, pp. 187–189, Aug. 1990.

- [26] M.-S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165–1181, Aug. 1999.
- [27] H. A. David, Order Statistics, 2nd ed. New York: Wiley, 1981.
- [28] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 2nd ed. Baltimore, MD: Johns Hopins Univ. Press, 1983.
- [29] S. Sandhu and A. Paulraj, "Space-time block codes: A capacity perspective," *IEEE Commun. Lett.*, vol. 4, no. 12, pp. 384–386, Dec. 2000.
- [30] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications—A key to gigabit wireless," *Proc. IEEE*, vol. 92, no. 2, pp. 198–218, Feb. 2004.
- [31] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels: A Unified Approach to Performance Analysis*, 1st ed. New York: Wiley, 2000.
- [32] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, Jun. 2002.
- [33] G. Fedele, "N-branch diversity reception of M-ary DPSK signals in slow and nonselective Nakagami fading," Eur. Trans. Commun., vol. 7, pp. 119–123, Mar.–Apr. 1996.
- [34] C. J. Chen and L. C. Wang, "Coverage and capacity enhancement in multiuser MIMO systems with scheduling," in *Proc. IEEE Globecom*, vol. 1, Dec. 2004, pp. 101–105.



Chiung-Jang Chen was born in Kaohsiung, Taiwan, R.O.C., in May 1971. He received the B.S. degree in electronics engineering from National Chiao-Tung University (NCTU), Hsinchu, Taiwan, R.O.C., in 1993, the M.E. degree in electrical engineering from National Taiwan University, Taipei, Taiwan, R.O.C., in 1995, and the Ph.D. degree in communication engineering from NCTU, Taiwan, R.O.C.

He is now with Chunghwa Telecom Laboratories, Chung-Li, Taiwan, R.O.C. His research interests include radio network resource management, MIMO

performance analysis, and cross-layer optimization for high-speed wireless networks.



Li-Chun Wang (S'92–M'95) received the B.S. degree from National Chiao Tung University, Hsinchu, Taiwan, R.O.C., in 1986, the M.S. degree from National Taiwan University, Taipei, Taiwan, R.O.C., in 1988, and the M.Sc. and Ph.D. degrees in electrical engineering from the Georgia Institute of Technology, Atlanta, in 1995 and 1996, respectively.

From 1990 to 1992, he was with the Telecommunications Laboratories of the Ministry of Transportations and Communications in Taiwan (currently, Telecom Labs of Chunghwa Telecom

Company). In 1995, he was affiliated with Bell Northern Research of Northern Telecom, Inc., Richardson, TX. From 1996 to 2000, he was with AT&T Laboratories, where he was a Senior Technical Staff Member in the Wireless Communications Research Department. Since August 2000, he has been an Associate Professor in the Department of Communication Engineering, National Chiao Tung University, Hsinchu, Taiwan, R.O.C. His current research interests are in the areas of cellular architectures, radio network resource management, and cross-layer optimization for high speed wireless networks. He holds three U.S. patents, and has one more pending.

Dr. Wang was a co-recipient of the Jack Neubauer Memorial Award in 1997, recognizing the best systems paper published in the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY. Currently, he is the Associate Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.